Bifurcations of dynamical systems workshop

9th - 12th February, 2022, Zagreb, Croatia

Minicourses

Peter De Maesschalck

Title: Singular perturbation theory

Abstract: TBA

Loïc Teyssier

Title: Inverse problems in planar dynamics: a constructive solution to the nonlinear Riemann-Hilbert problem

Abstract: TBA

Steffen Winter

Title: Minkowski content and fractal curvatures - a tool in dynamical systems?

Abstract: In this minicourse we give some introduction to Minkowski contents and the related concept of fractal curvatures. We do not assume prior knowledge on singular curvature theory but will provide the required background material on curvature measures. We survey some of the known results on fractal curvatures with a focus on self-similar and self-conformal sets. We will also address the different techniques of proof that are available. Some attention will be given to the Minkowski content and what can be learned from viewing it as one of the fractal curvatures. In a last part the associated scaling exponents will be discussed together with some recent alternative approach to study them.

We hope that this can provide a base for some discussion on whether and how fractal curvatures or some related concept might be useful in the study of dynamical systems. Some starting point on fractal curvatures is [1, Chapter 10] or [2]. Further references will be provided during the lectures.

References

[1] J. Rataj, M. Zähle, Curvature Measures of Singular Sets, Springer Monographs in Mathematics, https://doi.org/10.1007/978-3-030-18183-310

[2] Steffen Winter: Curvature measures and fractals, Diss. Math. 453 (2008), 1-66

Talks

Vlatko Crnković

Title: TBA

Abstract: TBA

Martin Klimeš

Title: Classification of germs of parabolic reversible diffeomorphisms of $(C^2, 0)$ with a first integral of Morse type

Abstract: A germ of analytic diffeomorphism of $(\mathbb{C}^2, 0)$ is reversible if it is conjugated to its inverse by an analytic involution. It is parabolic if some of its iteration is tangent to the identity. The talk is about analytic classification of such diffeomorphisms with respect to conjugation under an additional condition on existence of an analytic first integral of Morse type. The obtained description is a generalization to a higher dimension of the Birkhoff, Ecalle and Voronin modulus of parabolic diffeomorphisms of $(\mathbb{C}, 0)$. A particular motivation comes from a problem of Moser and Webster of

normal forms of certain CR-singularities of real-analytic surfaces in $\mathbb{C}^2$. We solve this problem for holomorphically flat surfaces (those contained in a real hyperplane) in the, so called, exceptional hyperbolic case. The talk is based on a joint work with Laurent Stolovitch.

Renato Huzak

Title: Fractal analysis of slow-fast systems

Abstract: In our talk we present a fractal analysis of canard cycles and slow-fast Hopf points in 2-dimensional singular perturbation problems under very general conditions. Given a slow-fast system, we generate a sequence of real numbers using the so-called slow relation function and compute a fractal dimension of that sequence. Then the value of the fractal dimension enables us to determine the cyclicity and bifurcations of canard cycles in the slow-fast system. We compute the fractal dimension of a slow-fast Hopf point depending on its codimension. Our focus is on the box dimension, one-sided dimensions and the fractal zeta-function. We also find explicit fractal formulas of Cahen-type for the computation of the above fractal dimensions and use them to detect numerically the number of canard limit cycles.

Pavao Mardešić

Title: Darboux relative exactness and pseudo-abelianintegrals

Abstract: I will present results of a joint work with Colin Christopher.

We study integrable systems in the plane $\mathbb{C}^2$ having a Darboux first integral $F = \prod_{j=0}^{\ell} f_i^{\lambda_i}$, $f_i \in \mathbb{R}[z]$, $\lambda_i \in \mathbb{R}$, and their polynomial deformations.

$$\frac{dF}{F\prod_{j=0}^{\ell} f_i} + \epsilon\eta = 0.$$

The displacement function along a cycle $\gamma(t)$ in $F^{-1}(t)$ for this deformation $\Delta_\epsilon(t)$ is of the form

$$\Delta_\epsilon(t) = \sum_{i=1}^{\infty} \epsilon^i M_i(t).$$

The functions M_i are called Melnikov functions and the first non-identically zero function M_i essentially controls the creation of limit cycles in the system. It is known that M_1 is given by the pseudo abelian integral $\int_{\gamma(t)} \eta$

We introduce a notion of Darboux relatively exact forms and prove that under generic hypothesis on the first integral F, if $M_1 = 0$, then the form η is Darboux relatively exact. We give an algorithm for calculating the first non-zero Melnikov function.

These results generalize previous results of Ilyashenko and Françoise given for deformations of integrable systems with polynomial first integral.

Dino Peran

Title: Normal forms for logarithmic transseries and Dulac germs

Abstract: We obtain short normal forms for logarithmic transseries by using fixed point theorems and solving various formal differential equations on appropriate spaces of logarithmic transseries. Therefore, normalizations are given as limits of Picard sequences in appropriate formal topologies. We apply these formal results to find analytic normal forms of (strongly) hyperbolic Dulac germs on standard quadratic domains. These results can be seen as generalizations of the classical Koenigs Theorem and the Böttcher Theorem.

Joint work with M. Resman, J.-P. Rolin and T. Servi

Goran Radunović

Title: Fractal zeta functions of orbits of parabolic diffeomorphisms

Abstract: We study the fractal zeta functions of orbits of parabolic diffeomorphisms. These zeta functions admit a meromorphic extension to the whole complex plane and their poles, also called the complex dimensions of the given orbit, are considered as the fractal footprint of the diffeomorphism since its formal class can be extracted from it. Moreover, the complex dimensions are strongly related to the generalized asymptotics of the orbit's tube function, i.e., the length of its epsilon neighborhood. Joint work with P. Mardesić and M. Resman.

Maja Resman

Title: Analytic invariants of Dulac germs

Abstract: In this talk we discuss different types of analytic classifications (ramified and non-ramified) of parabolic (tangent to the identity) Dulac germs. Dulac germs are germs on sufficiently big complex domains that admit a certain power-logarithmic asymptotic expansion, and they appear as first return maps of hyperbolic polycycles of planar systems. In case of ramified classification, we show a similar formal and analytic classification result as for regular germs that expand in Taylor series.

Domagoj Vlah

Title: Reconstruction of Incomplete Wildfire Data using Deep Generative Models and high-performance GPU computing

Abstract: We present our submission to the Extreme Value Analysis 2021 Data Challenge in which teams were asked to accurately predict distributions of wildfire frequency and size within spatio-temporal regions of missing data. For the purpose of this competition we developed a variant of the powerful variational autoencoder models dubbed the Conditional Missing data Importance-Weighted Autoencoder (CMIWAE). Our deep latent variable generative model requires little to no feature engineering and does not necessarily rely on the specifics of scoring in the Data Challenge. It is fully trained on incomplete data, with the single objective to maximize log-likelihood of the observed wildfire information. We mitigate the effects of the relatively low number of training samples by stochastic sampling from a variational latent variable distribution, as well as by ensembling a set of CMIWAE models trained and validated on different splits of the provided data. The presented approach is not domain-specific and is amenable to application in other missing data recovery tasks with tabular or tensor-shaped information conditioned on auxiliary information. Also, our approach is highly parallelizable and greatly benefits from a multi-GPU high-performance computing environment.

This is a joint work with Tomislav Ivek.

Vesna Županović

Title: Fractal analysis of bifurcations

Abstract: In this talk will I give the initial results concerning analysis of ε-neighborhoods of orbits of dynamical systems The idea comes from the fractal geometry, while the motivation comes from the 16th Hilbert problem. It is of interest to determine how many limit cycles can bifurcate from a given limit periodic set in a generic unfolding. The cyclicity is classically obtained by studying the multiplicity of fixed points of the Poincaré map. We establish a relation between the cyclicity of a limit periodic set of a planar system and the leading term of the asymptotic expansion of area of ε-neighborhoods of the Poincaré map of an orbit. A natural idea is that higher density of orbits reveals higher cyclicity. The box dimension could be read from the leading term of the asymptotic expansion of area of ε-neighborhood. In this talk I will concentrate on weak focus as a simplest case for the study.